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Abstract

The analysis in this paper is based on the common observation that many nonlinear dynamic processes may in fact be approximately linear over wide ranges of the economically relevant state space and hence over long periods of time. Only when state variables move into particular regions of the phase space may nonlinear reactions become apparent and perhaps only then, statistically detectable. Standard *unconditional* methods of testing linearity do not seem to have recognised the potential importance of this observation, that a given sample may not be particularly informative regarding nonlinearity since it may be only occasionally important and not uniformly represented throughout a given observation period. This, we believe, could be one reason why evidence for nonlinearity in the conditional mean has been difficult to find in economic time series. We argue that an explicitly *conditional* approach to testing linearity in which the *metric* used in inference incorporates any potential ancillary information reflecting the statistical curvature of the underlying data generation process may provide a more suitable framework for the detection of nonlinearity in economic time series.

We first put forward theoretical arguments for adopting a conditional approach to testing linearity and discuss the difficulties in following this suggestion through in applied work. Then, in an attempt to explicitly investigate the periodic importance of nonlinearity, we apply a set of linearity tests recursively to both monthly and weekly observations on the US Dollar/UK Pound Sterling spot exchange rate over the period 1973–1990. Our main interest lies in the detection of nonlinearity in the *first moment* of the data but we are also concerned that misspecification of the first moment may lead to error specifications that imply ARCH type processes and so also consider tests for conditional heteroskedasticity. The different tests we employ provide different indications of nonlinearity over different levels of temporal aggregation and under different transformations of the data but the overwhelming conclusion is clearly in favour of the *periodic* but not the continuous importance of nonlinear effects in the first moment of the data. The weekly series display considerably more evidence of nonlinearity than the monthly data and GARCH(1,1) residuals show the least evidence of nonlinearity both in first moment and naturally in the second moment, with random walk residuals clearly indicating model misspecification. Those periods in which the recursive tests indicate nonlinearity are briefly compared with an historical event analysis in an attempt to identify potential behavioural causes for the deviations from linearity. Some of these episodes appear to be associated with public reversals of government policy and intervention in the market for foreign exchange but others would seem to have no obvious economic cause. We also find little association between periods of first moment nonlinearity and periods of high volatility. Exploring such evidence for *periodic* nonlinear effects in the first moment of financial data further might aid the development of improved models of behaviour in financial markets.

1 Introduction

Given that there is no doubt that much of the physical world is characterised by nonlinearity it would indeed be surprising if economic behaviour were any different. The real question is surely not whether nonlinearities ultimately describe both the economic and physical worlds but whether linear models provide *adequate* approximations that are able to capture observed behaviour and whether standard statistical methods are appropriate to detect nonlinearity and provide valid inference when it is important.

In this paper we adopt a somewhat different position with regard to the detection of nonlinearity in economic time series than has been taken in much of the previous literature. We are motivated by the observation that many models with globally complex nonlinear behaviour can display approximate linearity over much of their *economically meaningful* phase space and only periodically exhibit nonlinearity, perhaps when the state variables pass through particular ranges. As econometricians there is a need to consider how to detect evidence for nonlinearity of this form where the "signal" may be only intermittent and not uniformly present in the available sample. Standard unconditional inference techniques may not be suitable when the nonlinear signal is weak in this sense. As economists, there is a need to build structural models that match the empirical evidence and to assess whether volatile markets, where behaviour is poorly explained by linear models such as those for foreign exchange, spend a significant portion of their time within nonlinear ranges or whether there are perhaps natural economic regulators that prevent the market entering nonlinear or chaotic phases.

Few nonlinear structural models of exchange rates have been specified and although standard linear specifications are generally recognised not to perform well, unambiguous evidence in favour of nonlinearity in the first moment of exchange rate data has been illusive with, it would seem, as much research accepting nonlinearity as rejecting; see Guarda and Salmon (1993). On the other hand, the one class of models, based on ARCH processes, that has gathered considerable empirical support and exhibits periodic volatility or nonlinearity in the *second moment* does not appear to completely capture all the systematic behaviour found in financial time series and exchange rates. Given that misspecified first moment behaviour may lead to ARCH type processes in the second moment it would seem necessary to rigorously examine the first moment specification before adopting an ARCH specification for the second moment. Moreover even if economic behaviour did imply periodic nonlinearity through the second moment it might perhaps be expected that such behaviour would also periodically affect the first and perhaps higher moments of the data. A broader class of nonlinear dynamic models might be called for to account for observed behaviour in *both* the first and second moment of financial data.

In what follows we seek to contribute to this process by questioning the statistical methodology that is traditionally adopted for testing linearity in economic time series when nonlinearity may only be *periodically* important in the data and by providing some, apparently fairly clear, evidence for periodic nonlinearity in the first moment of foreign exchange data. While we accept that arbitrage considerations might generally rule out continuous first moment predictability it may not at isolated periods. Moreover the expected equilibrium return process may deviate periodically from constancy leading to periodic non random walk behaviour in efficient exchange rates, see Levich (1979).

We start in Section 2, by briefly discussing the statistical issues raised in the detection of nonlinearity and emphasise the difference between the use of conditional and unconditional inference in this context. Many, apparently linear, econometric models are statistically curved in the sense of Efron (1975) and this forces an important distinction between conditional and unconditional inference techniques to be drawn. While we are able to discuss and demonstrate the importance of the difference between unconditional and conditional metrics in a simple statistically curved example, the determination of appropriate, approximate or exact, ancillary statistics on which to condition when testing linearity will in general depend on the specific statistical model under consideration and represents an important unresolved research area. This discussion is therefore indicative of how we feel testing for linearity might perhaps be developed in the future and may suggest one reason why clear statistical evidence for nonlinearity in the first moment of economic time series has often been difficult to obtain.

Recognising the arguments for conditional inference, we follow a conditional philosophy in the empirical analysis by applying a recursive approach to testing linearity in foreign exchange data. Section 3 describes the eight tests that are applied to four different series based on the Pound Sterling/US Dollar spot exchange rate: the raw data, the natural logarithms of the raw data, the log differences and the standardised residuals from a GARCH(1,1) model fitted to the log differences. The analysis in Section 4 is based on the use of the full-sample and is carried out at both monthly and weekly frequencies. In common with other research we find ambiguous results from this full sample analysis, but in Section 5 the same tests are applied recursively, providing, what we interpret as, clear evidence in favour of nonlinearity in the first moment at particular periods within the sample if not over the sample when taken as a whole. We also find, somewhat to our surprise, little association between periods of high volatility and those indicating first moment nonlinearity. Section 6 concludes, suggesting that exchange rate models should allow for periodic nonlinear reactions in both the first and second moment and that future testing of linearity may benefit from adopting an explicitly conditional statistical framework.

2 A Conditional Approach to the Detection of Nonlinearity

Granger and Terasvirta (1993) have recently provided a detailed discussion of a number of tests for linearity which can be seen to fall into two main classes; those for which a specific nonlinear alternative is specified and "general" tests for which no particular alternative is identified. Where specific restrictions for linearity can be identified it is natural that standard likelihood based methods are employed, through the use of either the Likelihood Ratio, Wald or Lagrange Multiplier (LM) tests. Wald tests seem to particularly inappropriate for testing linearity given their general lack of invariance to the algebraic form that a nonlinear restriction function may take, see for instance, Gregory and Veall (1985), Critchley, Marriott and Salmon (1993). However a critical statistical issue when testing linearity is not simply a question of the lack of invariance of a particular statistic but the impact that the statistical curvature of the underlying statistical model has on inference regarding linearity. This issue in its simplest form concerns the choice of the norming metric in a test statistic[1].

As Granger and Terasvirta show, a number of linearity tests in fact correspond to LM tests and this is natural given that the LM approach only exploits restricted parameter estimates and that the same test may be sensitive to a range of nonlinear alternatives since an LM statistic does not exploit information about the precise form of the alternative. Davidson and MacKinnon (1983) and Bera and McKenzie (1986) have explored, through monte carlo analysis, the behaviour of a number of different forms that an LM statistic may take given alternative choices that can be made for the norming metric in the test statistic. Different finite sample and robustness properties follow for these statistics, but in each case, in their examination of the issue, the norming metric was chosen so that it converged to the *unconditional* Fisher Information matrix. The question of constructing conditional score statistics where the norming metric corresponds to the conditional variance of the score, given some suitably defined ancillary statistic has not been addressed and is essentially the issue we wish to raise in the context of testing for linearity.

Severini (1990) has recently considered the conditional properties of unconditional likelihood based tests. He showed that the asymptotic conditional size of these tests in a one parameter case corresponds with the nominal size provided the *observed* Fisher Information matrix is used in the variance estimate. If the expected Fisher information matrix is used instead then the conditional size of the tests varies with the observed value of the (local) ancillary statistic which in turn depends on the statistical curvature of the model under investigation. Hence when statistical curvature is large there is a clear argument in favour of using conditional inference but more generally the indication from Severini's results is that the use of the observed information matrix forms of the likelihood based tests will often insulate the results of inference from the effects of curvature. The wide divergence between the conditional and unconditional levels of these tests in curved statistical models was also apparent in the simulations carried out by Efron and Hinkley(1978).

Efron(1975) defined statistical curvature at θ of a one parameter family of density functions as follows;

$$\gamma_\theta \equiv \left(\frac{|M_\theta|}{i_\theta^3}\right)^{1/2}$$

where i_θ represents the expected Fisher information per observation and M_θ is the covariance matrix of the observed score and Hessian of the log likelihood function, $\left(\dot{l}_\theta, \ddot{l}_\theta\right)$. As such it describes the standard notion of the geometric curvature of a line (representing the parametric statistical family of distributions of interest) in some space of all suitably defined distributions as the rate of change of direction with respect to arc length. In other words how fast the score statistic changes as θ moves through its range. Efron's main concern was to develop a measure of how far a given statistical family was from the exponential family since one parameter exponential families are known to have good statistical properties. In particular, in this case, the MLE is known to be sufficient and locally most powerful tests are also uniformly most powerful. Essentially these properties follow as the exponential family can be viewed, in Efron's measure of curvature, as being flat or described by a straight line in the space of all distributions. Statistical curvature is then zero everywhere and linear methods of statistical analysis, in other words those based on linear approximations to the log likelihood function, work well. Efron's argument was then that a large value of statistical curvature would imply that these properties would break down and in particular that locally most powerful tests would have poor operating characteristics and that the variance of the MLE would exceed the Cramer-Rao lower bound in proportion to γ_θ^2. Under repeated sampling Efron's measure of curvature will generally go to zero with sample size at a rate of $n^{1/2}$ and so asymptotically the effects of curvature will generally decay but in finite samples curvature depends on the particular model and the sample size. We also note that a distinction should be drawn between *parameter effects* curvature and *intrinsic curvature*, see for instance Seber and Wild (1989). Parameter effects curvature can be removed or reduced by reparametrising the statistical model whereas intrinsic curvature cannot and it is intrinsic curvature that drives the wedge between conditional and unconditional inference techniques.

Most nonlinear dynamic models will be statistically curved and indeed so will many linear dynamic models. For instance an AR(1) model has Efron curvature given by $\frac{2}{n}$ independently of the autoregressive parameter and an MA(1) with parameter θ has curvature given by ${}^{2(3-\theta^2)}/_{(1-\theta^2)n}$. So for instance if we crudely take Efron's value of one eighth as indicating when curvature will become important then this implies a sample size of at least 16 is needed with an AR(1) model to be able to ignore curvature effects but for an MA(1) with $\theta = 0.9$ a sample size of more than 185 is needed. Multivariate models may generate curved statistical families, for instance a "linear" AR(p) model represents a $((p+1)(p+2)/2, (p+1))$ curved exponential family and an ARMA(p,q), a $(n, p+q+1)$ curved exponential family,(see Ravishankar et al (1990) for a more detailed discussion).

The importance of these results for testing linearity lies in that since most dynamic models, whether linear or nonlinear, will be statistically curved potentially critically different results may arise from the use of conditional and unconditional metrics when testing for linearity. Standard forms of LM tests for linearity which employ unconditional metrics, through the use of the unconditional Fisher Information matrix, may seriously misrepresent evidence for nonlinearity in the data which may alternatively be detected using conditional metrics when the conditional variance of the score is calculated based on suitably defined ancillary statistics.

The basic argument for adopting a conditional framework for inference follows from the Conditionality Principle in statistics (see for instance Cox and Hinkley (1974)) which may be stated as follows;

The Conditionality Principle
Suppose $S = (\hat{\theta}, A)$ is minimally sufficient for θ and A is ancillary. Then inferences about θ are appropriately drawn in terms of the sampling behaviour of $\hat{\theta}$ under the assumption that A is constrained to the value 'a' observed in the sample.

The Conditionality Principle dates back to Fisher and his fundamental work in establishing the concepts of sufficiency and ancillarity and their role in inference, see Fisher

(1925),(1934),(1935). The essential difference between the conditional and the unconditional approaches to inference lies in establishing a frame of reference that is relevant to the context of application. The following two quotes may be useful in conveying the role of ancillary information in inference;

> *Ancillary statistics are only useful when different samples of the same size can supply different amounts of information and serve to distinguish those which supply more from those that supply less...* Fisher 1935

> *The idea of ancillary statistics simply tells us how to cut down the sample space to those points relevant to the interpretation of the observations we have...* Cox 1958

The mathematical analysis that we employ when determining the properties of tests and estimators rests on an *a priori* assumption that there will be some equivalence between the particular sample available and the assumed population. With frequentist inference we embed the statistical problem *ex ante* in a sampling framework by assuming that the random variables are, for instance, independent and identically distributed and follow some prescribed distribution. A set of assumptions which, although standard, is not innocuous since frequentist probability calculations such as the construction of a confidence interval have the interpretation of covering the true value , say, 95% of the time within a set of replications defined by these assumptions. A conditional frame of reference restricts the set of replications to that determined by the given value of the ancillary statistic which essentially provides information as to the experimental design of the observed sample or the adequacy of the assumed model given the available sample. The implicit methodological assumption that is at question is then whether the pre-data mathematical calculations under the assumed statistical model are relevant to post-data inference given the particular realisation of the data.

This question of setting the appropriate frame of reference for econometric inference concerns the selection of appropriate tools for inference, in other words which metric to use, conditional or unconditional and if conditional on what ancillary statistic should conditioning be taken. In particular in the context of testing linearity if statistical curvature is important and nonlinearity is not uniformly represented in the sample then the use of an unconditional metric may not provide *relevant* inference. Given the inability in economics to generate data from a well designed experiment economic data is potentially weak in that there may be situations where the available sample simply does not provide an adequate representation of the feasible range of behaviour implied by the true data generation process. The possibility exists for a true model to fail to be supported by a given data set in that some aspect of behaviour implied by the economic model is simply not observed in the available data. A situation which could potentially lead to the false statistical rejection of the true model. The relationship between the design of the available sample and the assumed statistical model should be explicitly recognised in the tools for inference by conditioning on ancillary statistics that capture the appropriate frame of reference for inference. Notice that this issue concerns not only the potentially limited information content of the data but the mathematical forms of test statistics that are employed to assess this information. The emphasis on the unconditional approach to likelihood based inference within econometrics has led to a situation in which we have developed relatively few techniques to explore this issue where perhaps we should not reject the model but perhaps instead " reject the data "!

A major difficulty in following through these arguments for adopting a conditional framework for inference lies in isolating suitable exact and unique ancillary statistics on which to condition. One common definition of an ancillary statistic is as follows;

Ancillary Statistic

If the minimal sufficient statistic , S , has larger dimension than the unknown parameter θ then A is called an ancillary statistic if we are able to write S=(T,A), where A has a marginal distribution which does not depend on θ. T is often referred to as conditionally sufficient since it may used as a sufficient statistic conditionally on A.

Given that the Maximum Likelihood estimate will in general be a function of the conditionally sufficient statistic it can be seen, from this definition that, ancillary statistics capture that part of the total information that is lost when the sample is reduced to the maximum likelihood estimate when it is not itself sufficient. This information is lost when unconditional inference is used but may be recovered by conditioning on the ancillary statistic which then serves to set the relevant frame of reference within which to conduct inference on the MLE and captures the adequacy of the assumed model's assumptions given the available sample. One intuitive explanation for the role of ancillary statistics lies in that they may contain information on the characteristics and shape of the likelihood function beyond simply the position of the MLE and hence to a first order approximation the Hessian or the observed information matrix will often provide a relevant ancillary and in fact a conditional precision measure for maximum likelihood inference. For many models however, where the likelihood surface is irregular and locally non-quadratic around the MLE, the use of the observed information matrix cannot be guaranteed to provide an exact ancillary statistic and may only form an approximate ancillary [2].

Since it is impossible to develop these arguments in detail here we shall make our comparison between conditional and unconditional inference when testing linearity on the difference between the use of the observed information matrix, I, and the expected information matrix, $\Im$, although, as we have stressed, a rigorous analysis would require the development of conditional metrics, $\Im(\hat{\theta}|A = a)$, that recognise the higher order effects of statistical curvature on inference in any particular case. These two measures of precision can obviously be quite different. One important observation of Efron and Hinkley(1978) is that for large n the difference between I and $\Im$ is, under certain conditions, given by

$$\sqrt{n}\frac{I - \Im}{\Im} \sim N(0, \gamma_\theta^2)$$

where γ_θ^2, the *statistical curvature* of the model, varies with the parameter θ of the statistical model, and this result clearly indicates the intimate connection between curvature, information loss and the need for conditional inference. One natural approximate criterion for assessing the importance of conditional arguments can then be taken as the standardised ratio;

$$A_n = \frac{\sqrt{n}}{\gamma_\theta}\left(\frac{I}{\Im} - 1\right)$$

which serves as an approximate ancillary statistic and which will be asymptotically distributed as N(0,1). Given the approximate ancillarity of the observed information matrix, this statistic provides an indication of when the potential range of behaviour under the assumed model's assumptions is poorly represented in the available sample and hence when a significant difference between conditional and unconditional inference may arise. An obvious suggestion is then that it could be reported in applied econometric research to indicate at least the potential importance of conditioning.

The discussion above has very briefly emphasised general aspects of the distinction between conditional and unconditional inference but, as we have stressed, the feasibility of applying conditional arguments to testing linearity rests on the construction of relevant ancillary statistics. One obvious but invariably only approximate suggestion might be to use observed information matrix forms of LM tests although we suspect that further research into this question will lead to better approximate ancillaries and more appropriate conditional metrics when testing linearity[3]. However even without preicse indications of suitable ancillary statistics we can demonstrate how even approximate ancillary statistics can pick up particular sample information that affects inference regarding nonlinearity by calculating the conditional and unconditional precision measures under different sampling schemes in a simple monte carlo experiment.

In this experiment we generate different samples drawn from a nonlinear, statistically curved, bivariate regression model which have different information regarding the nonlinearity in the true underlying data generation process. Note that this data generation structure corresponds somewhat crudely, with the case made in the introduction for the periodic, but not uniform presence of nonlinearity in the first moment of economic data.

We consider a bivariate regression model of the form;

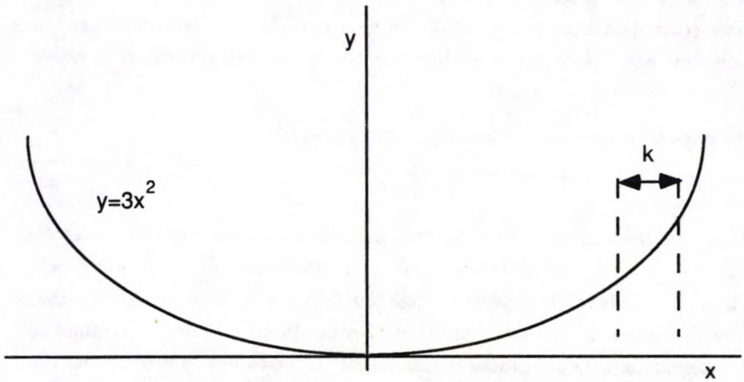
$$y_t = cx_t^2 + \varepsilon_t$$

where $\{x_t, \varepsilon_t\}$ are independent standard normal variables. This stochastic linear regression model is statistically curved and hence, from the arguments above, we can expect a distinction to arise between the use of conditional and unconditional inference that depends on the degree of statistical curvature. Let us also assume that the sampling structure determining the observations is constrained, as shown in Figure 1, in that observations on the regressor are drawn only from a fixed interval of width , k , around some fixed point, say x=3, so the mean of the standard normal, x_t, variable becomes 3.

The maximum statistical curvature in this example occurs at the origin and the sample observations are then drawn from a restricted, relatively linear part of the total potential reaction space where the statistical curvature is approximately half the value of that at the origin. The index, k , serves essentially as an ancillary statistic that indicates the sampling stratification and we are interested in how inference varies as this ancillary takes different values and more " nonlinear" information is incorporated in the sample. A small monte carlo study, with 200 replications of 50 observations each, was carried out on this model from which we compute the conditional and unconditional variances of the least squares estimate of c, using the inverse of the observed and expected information respectively, given our exact prior knowledge of the distribution of the regressor (non central χ^2), as well as the observed sampling variation of the parameter estimate drawn from the monte carlo experiments as the interval determined by the value of k increases in size from 0.05 to 5 in 100 steps.

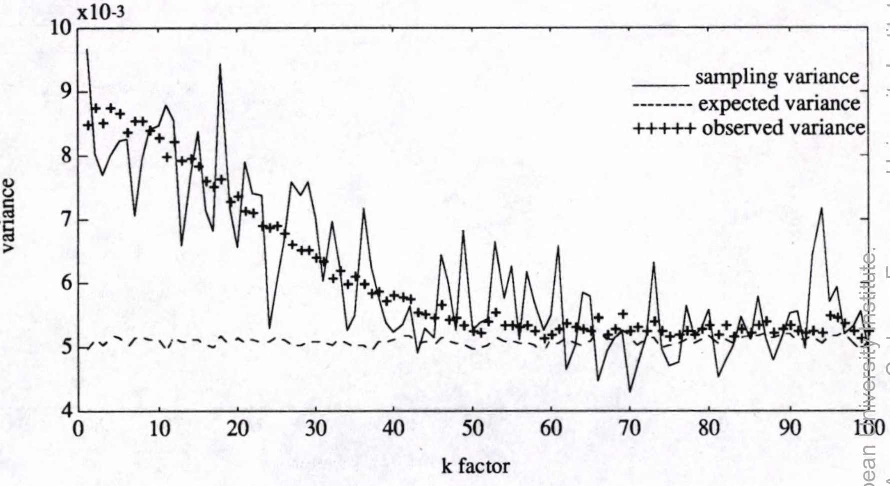
Figure 2 shows these three measures of precision as k varies. The observed variance determined by the standard least squares formula tracks the sampling variation relatively well and only after the sampling interval has stretched sufficiently and k reaches a

Figure 1: The Nonlinear Regression Surface



value of about 40 does the unconditional variance, calculated using the expected information matrix and the assumed distribution for the regressor, start to provide a reasonable approximation to the actual sampling variance of the estimate determined in the monte carlo. Notice in particular, how the unconditional estimate is insensitive to the sampling stratification, in that it does not reflect the way in which the information content of the sample changes with k. Both the sampling variance and the observed variance are higher than the expected variance, initially almost twice as large, until more evidence for nonlinearity becomes apparent in the sample as k increases. Notice also that the expected information is calculated based on the true nonlinear stochastic model that we in fact know in this case is generating the data. However when testing linearity using LM tests based on restricted linear models the null on which the expectation is formed will be the *linear* model hence the question arises as to whether the unconditional metric is relevant and whether a conditional metric might provide more appropriate inference. We are also able to compute the A_n statistic given above to determine if there is a statistical difference between the observed and expected information measures. Figure 3 plots this statistic again as k varies from 0.05 to 5. On a standard one sided 95% significance test we can see that the difference is significant up to about the value of k=25.

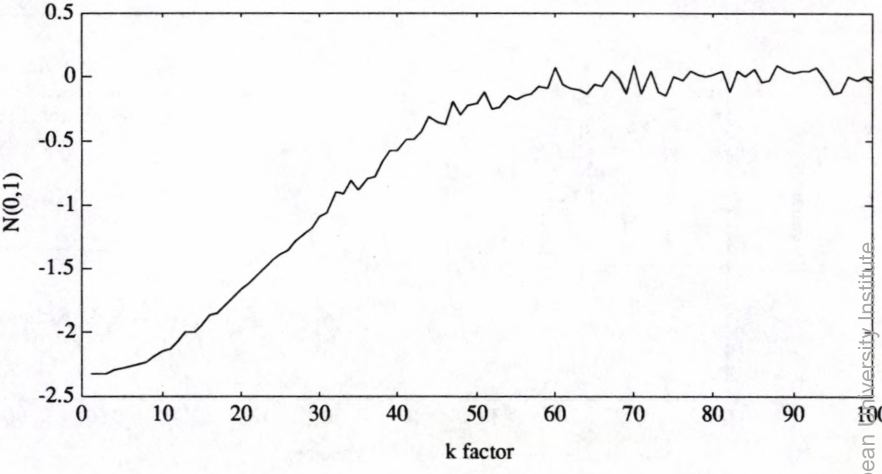
Figure 2: Conditional and Unconditional Precision Measures



What this simple exercise has shown is the fairly obvious point that different metrics may imply substantially different measures of precision when evaluated with the same data but more importantly that inference given the particular information content of an available sample when testing for linearity could be quite different when conditional and unconditional precision measures are employed.

The general argument for using conditional inference when testing linearity has been made but the question of how to construct suitable exact or approximate ancillary statistics for this purpose remains to be developed. We now turn to consider explicitly testing for linearity keeping in mind the arguments for conditional inference made above and will in particular, consider the use of recursive tests where the estimated metric used at each sample point changes as the sample size is increased and therefore inference is conditional on different sample information at each point but the mathematical form of the metric does not change. While we are clearly not formally using conditional metrics in the recursive LM tests below we hope in this way to at least construct a better inferential framework for detecting nonlinearity when it may not be uniformly represented in the sample.

Figure 3: Ancillarity Test for the Significance of Conditional Inference as k varies



3 Testing for Linearity

Testing for linearity has generated a range of alternative statistics, perhaps starting with the work of Ramsey (1969); a recent survey and discussion can be found in Granger and Terasvirta (1993). We shall explicitly build on the paper by Lee, White and Granger (1993), concentrating on testing for linearity in the mean and extending their work by applying recursive forms of the tests described below[4]. They define a process $\{y_t\}$ to be *linear in mean* with respect to the information set X_t of dimension k (which may, but need not, contain lagged values of y_t) if there exists a k-by-one vector of coefficients θ^* such that

$$P\left[E(y_t \mid X_t) = X_t'\theta^*\right] = 1 \text{ for some } \theta^* \in \mathcal{R}^k$$

The alternative hypothesis of interest in this case is that the process $\{y_t\}$ is nonlinear in mean with respect to X_t, so that

$$P\left[E(y_t \mid X_t) = X_t'\theta^*\right] < 1 \text{ for all } \theta^* \in \mathcal{R}^k$$

We shall also adopt the same strategy of testing for *neglected nonlinearity* so that the tests will, where appropriate, be applied to the residuals of an AR(p) model which we presume has extracted as much linear structure as possible from the data. The number of lags p is either chosen using the AIC criterion or the standard Box-Jenkins identification procedure.

3.1 The Keenan Test

The Keenan (1985) test is based on a truncated Volterra series expansion. It usually restricts the information set X_t to the past p observations of y_t so that it regresses y_t on $(y_{t-1}, y_{t-2}, ..., y_{t-p})'$ although it can be generalized to include other explanatory variables. It consists of the following steps:

(i) Regress y_t on X_t linearly $\qquad y_t = X_t'\theta + e_t$
producing a vector of coefficient estimates $\qquad \hat{\theta}$
from which one can generate fitted values $\qquad \hat{y}_t = X_t'\hat{\theta}$
and estimated residuals $\qquad \hat{e}_t = y_t - \hat{y}_t$

(ii) Regress the squares of the fitted values $\hat{y}_t^2$ on X_t $\qquad \hat{y}_t^2 = X_t'\lambda + \varepsilon_t$
and save the estimated residuals $\hat{\varepsilon}_t$

(iii) Regress the residuals $\hat{e}_t$ on the residuals $\hat{\varepsilon}_t$ $\qquad \hat{e}_t = \hat{\varepsilon}_t'\delta + \nu_t$
and save the estimated residuals $\hat{\nu}_t$

(iv) Test the null hypothesis $H_0 : \delta = 0$ using the Keenan test statistic:

$$\frac{\hat{e}'\hat{\varepsilon}(\hat{\varepsilon}'\hat{\varepsilon})^{-1}\hat{\varepsilon}'\hat{e}}{\hat{\nu}'\hat{\nu}/(n-2p-2)}$$

where $\hat{e} = (\hat{e}_1, ..., \hat{e}_n)'$, $\hat{\varepsilon} = (\hat{\varepsilon}_1, ..., \hat{\varepsilon}_n)'$ and $\hat{\nu} = (\hat{\nu}_1, ..., \hat{\nu}_n)'$.

The Keenan statistic has an $F(1, n-2p-2)$ distribution under the null hypothesis of no nonlinearity, where p is the number of explanatory parameters used (usually the number of lags in the autoregression). The Keenan test checks whether the squared fitted value $(\hat{y}_t^2 = \hat{\theta}' X_t X_t'\hat{\theta})$ has any additional explanatory power over the linear model. It has an LM interpretation similar to the RESET tests below.

3.2 The Ramsey RESET Test

The Ramsey (1969) Regression Error Specification Test can be seen as a generalization of the Keenan test since it examines the explanatory significance of higher powers of the fitted values $\hat{y}_t$ for y_t against:

(i) Regress y_t linearly on X_t $\qquad y_t = X_t'\theta + e_t$
and save the estimated residuals $\hat{e}_t$ and fitted values $\hat{y}_t$

(ii) Regress y_t on k powers of the fitted values $\hat{y}_t$ where k can be set at any level:

$$y_t = X_t'\theta + c_2\hat{y}_t^2 + c_3\hat{y}_t^3 + ... + c_h\hat{y}_t^k + \nu_t$$

and save the estimated residuals $\hat{\nu}_t$

(iii) Test the null hypothesis $H_0 : c_2 = c_3 = ... = c_h = 0$ using the RESET statistic:

$$\frac{(\hat{e}'\hat{e} - \hat{\nu}'\hat{\nu})/(k-1)}{\hat{\nu}'\hat{\nu}/(n-k)}$$

This form of the RESET test follows an $F(k-1,\, n-k)$ distribution under H_0. Another form suggested by Thursby and Schmidt (1977) was found to have superior power against nonlinear alternatives and can be obtained by regressing the error term from the autoregression against powers of the lagged dependent variables:

(i) Regress y_t linearly on X_t $\qquad y_t = X_t'\theta + e_t$
and save the estimated residuals $\hat{e}_t$

(ii) Regress the estimated residuals $\hat{e}_t$ on h powers of the explanatory variables X_t:

$$\hat{e}_t = X_t'\theta + X_t'^{(2)}\gamma_2 + X_t'^{(3)}\gamma_3 + ... + X_t'^{(h)}\gamma_h + v_t$$

and save the estimated residuals $\hat{v}_t$. Note that $X_t'^{(j)}$ denotes the vector containing the elements of X_t' each raised to the power j where $j = 2, ..., h$ and h is chosen at will (4 being the order recommended). Whereas in the basic RESET test c_h is a scalar, here γ_j is a different vector of k coefficients for each power from 2 to h.

(iii) Test the null hypothesis $H_0 : \gamma_2 = \gamma_3 = ... = \gamma_h = 0$ using the RESET2 test statistic:

$$\frac{(\hat{e}'\hat{e} - \hat{v}'\hat{v})/(h-1)}{\hat{v}'\hat{v}/(n-m-p-h)}$$

This modified RESET statistic follows an $F(h-1,\, n-m-p-h)$ distribution under the null where p is the order of the autoregression (dimension of X_t), h is the highest power to which X_t is raised and m is the number of coefficients estimated in the auxiliary equation (hxp). Both these RESET tests have LM interpretations and in common with the Keenan test are sensitive to departures from linearity in the mean.

3.3 The Tsay Test

Like the Keenan test, the Tsay (1986) test starts by regressing y_t on its past values so that X_t consists of $(y_{t-1}, ..., y_{t-p})'$. However, the Tsay test truncates the Volterra series expansion at a higher order and hence it features a more complicated second stage that involves the cross-products of past observations $y_{t-j}y_{t-k}$:

(i) Regress y_t on X_t linearly $\quad y_t = X_t'\theta + e_t$
and save the estimated residuals $\hat{e}_t$

(ii) Compose the vector P_t where $\quad P_t = vech(X_t'X_t)$
Since $vech(\cdot)$ denotes the half-stacking vector operator P_t consists of a $p(p+1)/2$ element vector containing the elements of the lower triangular part of $X_t'X_t$. Thus for each observation of y_t there corresponds vector P_t whose elements are the unique cross-products of the last p observations of y in other words $y_{t-i}y_{t-j}$ for i, j=1, ...,p where $j \geq i$.
Regress this vector P_t on the explanatory variables X_t $\quad P_t = X_t'\lambda + \varepsilon_t$
and save the estimated residuals $\hat{\varepsilon}$

(iii) Regress the estimated residuals $\hat{e}_t$ on $\hat{\varepsilon}_t$ $\quad \hat{e}_t = \hat{\varepsilon}_t'\delta + \nu_t$
and save the estimated residuals $\hat{\nu}_t$

(iv) Test the null hypothesis $H_0 : \delta = 0$ using the Tsay test statistic:

$$\frac{\hat{e}'\hat{\varepsilon}(\hat{\varepsilon}'\hat{\varepsilon})^{-1}\hat{\varepsilon}'\hat{e}/m}{\hat{\nu}'\hat{\nu}/(n-p-m-1)}$$

where again m is the number of coefficients estimated in the auxiliary equation (i.e. step (iii)) so that $m = p(p+1)/2$.

The Tsay statistic has an F(m,$n-p-m-1$) distribution under the null. It tests the forecasting ability gained by including product terms such as $y_{t-i}y_{t-j}$ or y_{t-j}^2. Again it has an LM interpretation as shown in Granger and Terasvirta(1993) and is sensitive to departures from linearity in the mean.

3.4 The McLeod-Li Test

This test is based on the principle that if the residuals follow a linear *iid* process the cross-product of their squares should have the same correlation structure as the square of their cross-products: $corr(y_t^2, y_{t-k}^2) = [corr(y_t, y_{t-k})]^2$ for all k. McLeod and Li (1983) apply a standard Box-Ljung portmanteau test for serial correlation to the squared residuals from a linear model to test for linearity.

(i) Regress y_t linearly on X_t $$y_t = X_t'\theta + e_t$$
and save the estimated residuals $\hat{e}_t$.

(ii) Calculate the autocorrelation function of the squares of the estimated residuals $\hat{e}_t^2$ up to any order m:

$$\hat{r}(i) = \frac{\sum_{t=i+1}^{m}(\hat{e}_t^2 - \hat{\sigma}^2)(\hat{e}_{t-i}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{n}(\hat{e}_t^2 - \hat{\sigma}^2)^2}$$

where $\hat{\sigma}^2 = n^{-1}\sum_{t=1}^{n}\hat{e}_t^2$

(iii) Test the null hypothesis of linearity using the statistic:

$$n(n+2)\sum_{j=1}^{m}(\hat{r}(j))^2/(n-j)$$

Under the null hypothesis the McLeod and Li statistic tends to a $\chi^2(m)$ distribution where the parameter m is chosen at the time of application. This test can be seen an LM test against ARCH (see again Granger and Terasvirta) although designed as a "general" test for nonlinearity.

3.5 The Brock-Dechert-Scheinkman (BDS) Test

The BDS test developed in Brock, Dechert and Scheinkman (1987) may be applied to the residuals of a linear autoregression to check whether they are generated by an independent and identically distributed process. It was derived from consideration of deterministic chaotic processes and is based on the Grassberger-Procaccia correlation exponent. Consider a vector of m observations $Y_{q,r}$ which consists of a subsample of $\{y_t\}$ over the interval $t = q$ to $t = r$ where $q \geq 0$ and $r \leq n$. Then compare a pair of such m-dimensional vectors $Y_{q,r}$ and $Y_{u,v}$. They are said to be no more than ε apart if this is true for each pair of their corresponding elements:

$$\| Y_{q,j} - Y_{u,j} \| \leq \varepsilon, \text{ j=1,...,}m$$

The Correlation Integral is then defined

$$C_m(\varepsilon) = \lim_{T\to\infty} T^{-2}\{\text{number of pairs of vectors such that the above holds}\}$$

which is a measure of the number of pairs of m-vectors (subperiods of length m within the sample) that are within a distance ε from each other. The Correlation Exponent V_n given below is used to distinguish chaotic from stochastic processes.

$$V_n = \lim_{\varepsilon \to 0} \frac{\partial \log(C_m(\varepsilon))}{\partial \log(\varepsilon)}$$

If a process is really stochastic V_n will increase linearly in n. Conversely, if it really is chaotic $C_m(\varepsilon) = \varepsilon^V$ and V is independent of the sample size n. The BDS test statistic below is constructed on the Correlation Integral C_m:

$$BDS = \sqrt{n}[C_m(\varepsilon) - C_1(\varepsilon)^m]/\sigma_n$$

where σ_n is a complicated variance expression and the statistic follows a normal distribution under the null hypothesis. Note that the critical distance ε and the dimension m of the BDS test are to be chosen at the time of application. Hsieh and LeBaron (1988) establish by simulation that the test has good power against nonlinear alternatives, both stochastic and deterministic, and if the null of independence is rejected it may be because of chaos or a nonlinear stochastic alternative. In samples of size n=500, asymptotic normality seems accurate for ε between 0.5 and 1.5 times the standard deviation of the data and for dimensions m up to about 6. There appears to be no LM interpretation for the BDS test.

3.6 The ARCH-LM Test

We have also included the basic ARCH LM test in order to explicitly consider questions of both first and second moment nonlinearity in the data. In principle we could have attempted to insulate the first moment tests for linearity from ARCH effects by using heteroskedastic consistent forms of the test statistics but for similar reasons to those discussed by Lee, White and Granger did not do so. The presence of conditional second moment structure is critically dependent on conditional first moment structure and given the methodological issues involved we preferred to use the uncorrected forms. ARCH models, introduced by Engle (19983), describe series in which periods of high volatility alternate with periods of low volatility so that a large change is more likely to follow a large change and a small change is more likely to follow a small change. In its simplest form this may be written formally:

$$\begin{aligned} y_t &= X_t'\theta + \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t) \\ h_t^2 &= \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \dots + \alpha_p\varepsilon_{t-p}^2 \end{aligned}$$

Engle proposed a Lagrange Multiplier test of the null hypothesis that $\alpha_1 = \alpha_2 = ... = \alpha_p = 0$. The procedure is as follows:

(i) Regress y_t linearly on X_t $\qquad y_t = X_t'\theta + e_t$

and save the estimated residuals $\hat{e}_t$.

(ii) Regress the squares of the estimated residuals $\hat{e}_t^2$ on an intercept and p lagged values of $\hat{e}_t^2$.

$$\hat{e}_t^2 = \alpha_0 + \alpha_1\hat{e}_{t-1}^2 + \alpha_2\hat{e}_{t-2}^2 + ... + \alpha_p\hat{e}_{t-p}^2 + \varepsilon_t$$

and save the estimated residuals $\hat{\varepsilon}_t$.

(iii) Calculate the R^2 from the second regression and test the null hypothesis using the nR^2 statistic.

The ARCH-LM test follows a $\chi^2(p)$ distribution under the null hypothesis of no ARCH dependence. This test, and the McLeod and Li test explicitly examine evidence for nonlinearity in the second moment whereas the other tests we consider are concerned with nonlinearity in the mean.

3.7 The Neural Network Test

The neural network test for neglected nonlinearity as described in Lee, White and Granger (1993) is based on the premise that the observed "output" $f(X_t, \delta)$ can be decomposed into a linear part $X_t'\theta$ and a nonlinear part $\sum_{j=1}^{q} \beta_j\psi(X_t'\gamma_j)$ constructed from a neural network. Thus $f(X_t, \delta) = X_t'\theta + \sum_{j=1}^{q} \beta_j\psi(X_t'\gamma_j)$ and under the null hypothesis of linearity the β_j in the augmented neural network will be zero for all j. The neural network test then checks the hypothesis $\beta_j = 0, j = 1, ..., q$ for a particular choice of q and γ=(γ_0, γ_1,...,γ_q). The test has power whenever $\sum_{j=1}^{q} \beta_j\psi(X_t\gamma_j)$ is capable of extracting structure from the residuals of the linear regression $\hat{e}_t = y_t - X_t\hat{\theta}$.

The neural network test exploits the fact that under the null hypothesis of linearity $E\left[e_t \mid X_t\right] = 0$ with probability one. Therefore e_t is also uncorrelated with any measurable function of X_t, which we can denote $h(X_t)$, including the activations of the intermediate units $\tilde{X}_t'\gamma_j$. So an alternative form of the null hypothesis is:

$$E(h(X_t)e_t) = 0$$

In implementing the neural network test, one generates "test functions" of X_t choosing for $h(X_t)$ the activations of hidden units $\psi(X_t'\Gamma_j)$ for j=1,...,q where Γ_j are random column vectors generated independently of y_t and X_t and q is set at will. Denoting the vector of activations thus generated as $\Psi_t = (\psi(X_t'\gamma_1),..., \psi(X_t'\gamma_q))'$, the null hypothesis can be reformulated:

$$E(\Psi_t e_t) = 0$$

So that evidence of correlation of Ψ_t and e_t indicates that augmenting the linear network by including additional hidden units with nonlinear activations $\psi(X_t'\gamma_j)$ would improve forecasting performance. The neural network test is implemented as follows:

(i) Regress y_t linearly on X_t $\quad y_t = X_t'\theta + e_t$ and save the estimated residuals $\hat{e}_t$.

(ii) Choosing Γ independently of $\{X_t\}$ and $\{y_t\}$, form the vectors Ψ_j and regress $\hat{e}_t$ on X_t and Ψ_t $\quad \hat{e}_t = X_t'\delta + \Psi_t'\lambda + \varepsilon_t$ and save the estimated residuals $\hat{\varepsilon}_t$

(iii) test the null hypothesis using an nR^2 statistic

Under the null hypothesis of linearity in the mean, the neural network test asymptotically follows a $\chi^2(q)$ distribution.

4 Full Sample Analysis

We now apply the nonlinearity tests described above to the Spot US Dollar/UK Pound Sterling exchange rate. We adopt an AR(4) as a linear filter, so the null hypothesis is that the series under question is linear in the mean with respect to its last four lags. For the Keenan and RESET test this means that the matrix X_t of explanatory variables consists of a constant and four lags, but for the RESET2 test and the Tsay test this would cause multicollinearity. Therefore we exclude the constant when X_t is raised to a power for the RESET2 test[5] and also when the vector of unique cross-products P_t is constructed for the Tsay test. The tests are estimated with the following parameters: for the Keenan test p=5; for the RESET k=4; for the RESET2 h=4, p=4 and so m=16; and for the Tsay test p=4 and so m=10. For the McLeod-Li test m=50; for the BDS ε is equal to the standard deviation of the input series and m=6; and for the Neural Network test a network with two hidden units was estimated to provide the vector of activations Ψ_t.

Our monthly data covers the period January 1973 to June 1992 whereas the weekly data is over the shorter period 1st January 1973 to the 7th May 1990. Note that the weekly data was constructed by averaging over the available daily observations in the given week rather than just picking the Wednesday observation as is common practice. We consider four separate transformations of the series:

- The Raw Data: Pound Sterling against US Dollar
- The Natural Logarithms of the Data
- The Log Differences (Random Walk Residuals)
- The GARCH(1,1) Standardised Residuals

The Raw Data, is seasonally unadjusted and neither differenced nor detrended and the log differences are the equivalent of the residuals from a Random Walk (in logs) with a unit coefficient imposed on the lag and a zero intercept term. The fourth series represents the standardised residuals of a GARCH(1,1) model fitted to the log differences[6]. These residuals are standardised by centering on their mean and dividing each one by the corresponding conditional variance as estimated by GARCH using the method described in Calzolari and Fiorentini (1992).

Table 1: Descriptive Statistics

	Raw	Logs	Log Diffs	GARCH
Monthly £/$ Spot Rate				
Observations	236	236	235	232
Maximum	2.5760	0.9462	0.1034	0.0111
Minimum	1.0961	0.0917	-0.1128	-0.0126
Skewness	0.2777	-0.1232	0.0067	0.0351
Kurtosis	2.3890	2.6140	4.1510	3.5320
Normality	6.704	2.062	12.974	2.74
Weekly £/$ Spot Rate				
Observations	906	906	905	902
Maximum	2.5800	0.9476	0.0658	0.0578
Minimum	1.0630	0.0612	-0.0412	-0.0790
Skewness	0.2219	-0.1262	0.2366	-0.5339
Kurtosis	2.1053	2.3313	5.3982	10.501
Normality	37.63	18.54	222.6	2157

Table 1 reports the basic statistics for the series. Note immediately the difference between the monthly and weekly data; weekly data consistently deviates further from normality than its monthly counterpart (the table reports the basic skewness and kurtosis coefficients). Note that transforming the Raw Data by taking logs improves kurtosis for both monthly and weekly series, increasing it towards its normal value of 3.0; the Bowman Shenton normality test (see for instance, Kiefer and Salmon (1983)) is correspondingly reduced. When the logs are differenced in the third column, kurtosis practically doubles, increasing the normality statistic, dramatically in the case of the weekly data. However, after the GARCH model is fitted to the log differences kurtosis is reduced for the monthly series, reducing its normality test, while for the weekly series kurtosis is again dramatically increased and the normality test takes the value of 2157 with a critical value of 5.99; this is probably due to two significant outliers). This table suggests that the normality assumption can be questioned for all of these series except the logs and GARCH residuals at the monthly frequency, that weekly data probably deviates further from normality than monthly data and that the GARCH model is clearly better suited to the monthly series than to the weekly series.

Table 2: Autocorrelation Functions: Monthly Data

	Raw	Logs	Log Diffs	GARCH
£/$ Spot Rate				
$\rho(1)$	0.9769	0.9802	0.3708	0.3322
$\rho(2)$	0.9522	0.9563	0.0493	0.0682
$\rho(3)$	0.9282	0.9832	0.0295	0.0082
$\rho(4)$	0.9030	0.9079	-0.0260	-0.0321
$\rho(5)$	0.8789	0.8848	-0.0340	-0.0252
$\rho(6)$	0.8571	0.8618	0.0438	0.0785
$\rho(12)$	0.6960	0.7145	0.0709	-0.0871
Squared Series:				
$\rho(1)$	0.9736	0.9752	0.0545	-0.0507
$\rho(2)$	0.9463	0.9490	-0.0292	-0.0855
$\rho(3)$	0.9203	0.9238	0.1926	0.0031
$\rho(4)$	0.8931	0.8977	-0.0128	-0.0863
$\rho(5)$	0.8671	0.8728	-0.0213	-0.0506
$\rho(6)$	0.8441	0.8505	0.0260	-0.0543
$\rho(12)$	0.6742	0.6832	-0.0130	-0.0116

Table 2 presents the Autocorrelation Function of the monthly series. The strong autocorrelation frequently associated with a unit root in exchange rates is visible in the first two columns and while taking the logs has a very small impact, first differencing the logs eliminates most of the autocorrelation. Fitting the GARCH model to the log differences transforms the pattern of autocorrelation and if anything appears to increase its significance. The lower panel of Table 2 presents the Autocorrelation Function for the

Table 3: Autocorrelation Functions: Weekly Data

	Raw	Logs	Log Diffs	GARCH
£/\$ Spot Rate				
$\rho(1)$	0.9968	0.9969	0.2370	0.2647
$\rho(2)$	0.9938	0.9938	0.0168	0.1117
$\rho(3)$	0.9903	0.9905	0.0293	0.0939
$\rho(4)$	0.9866	0.9870	0.0935	0.1033
$\rho(5)$	0.9823	0.9831	0.0258	0.0301
$\rho(6)$	0.9780	0.9790	-0.0160	0.0009
$\rho(12)$	0.9506	0.9536	0.0113	0.0314
Squared Series:				
$\rho(1)$	0.9967	0.9968	0.1682	-0.0099
$\rho(2)$	0.9934	0.9937	0.0846	-0.0093
$\rho(3)$	0.9895	0.9900	0.1293	-0.0276
$\rho(4)$	0.9853	0.9860	0.1770	0.0050
$\rho(5)$	0.9805	0.9815	0.0442	-0.0362
$\rho(6)$	0.9756	0.9769	0.0370	-0.0234
$\rho(12$	0.9448	0.9479	0.1075	0.0399

squared values of each series. Again in this case, the ACF of the first two series takes the form of a gentle decline, the other two series producing ACFs that show little or no significance. Again, this indicates that taking log differences is successful in removing most of the second moment linear dependence. However, the GARCH residuals provide some indication of residual nonlinear dependence.

Table 3 presents the Autocorrelation function for the weekly data and their squared values. The results are much as for the monthly data except that the evidence of autocorrelation is considerably more pronounced except for the GARCH residuals.

Table 4 reports the full-sample results of the linearity tests run on the four basic monthly series. The double asterisk denotes significance at the 1% level. Note that each of the series provides some indication of nonlinearity except the GARCH standardised residuals. The Keenan, RESET, Tsay, McLeod-Li and Neural Net tests detect no evidence of nonlinearity for any of the series. However, the RESET2 test detects nonlinearity for both the Logs and the Random Walk residuals (i.e. log-differences). The BDS test rejects the null hypothesis of independence for the Raw Data, the logs and the log-differences but not for the GARCH residuals. Predictably, the ARCH-LM test is significant for the first three series and not for the GARCH(1,1) residuals; however, note that it is sharply reduced by the log-differencing transformation. So the conclusions from the full sample

Table 4: Linearity Tests: Monthly Data

Test	Raw	Logs	Log Diffs	GARCH	Dist.
£/$ Spot Rate					
Keenan	0.1878	0.1518	1.2348	0.2994	F(1,200)
RESET	0.5512	0.1100	1.0307	1.1362	F(3,200)
RESET2	2.2736	**4.6403	**4.9440	2.0527	F(3,200)
Tsay	0.3841	0.8317	1.6549	0.7439	F(10,200)
McLeod-Li	29.178	28.681	30.664	47.210	$\chi^2(50)$
BDS(6)	**69.381	**306.89	**-6.113	-0.757	N(0,1)
ARCH-LM	**219.69	**223.41	**17.492	5.6808	$\chi^2(4)$
Neural Net	1.1573	0.1543	0.1074	0.4635	$\chi^2(2)$

tests on the monthly data appear to coincide with accepted wisdom that a GARCH model seems to capture most of the structure from the original series although there is some weak evidence of residual autocorrelation in both the levels and the squared GARCH residuals from Table 2. Otherwise the indicated misspecifications relate to a lack of independence and ARCH structure in the other three series.

Table 5: Linearity Tests: Weekly Data

Test	Raw	Logs	Log Diffs	GARCH	Dist.
£/$ Spot Rate					
Keenan	0.4916	0.9838	0.2368	0.9717	F(1,1000)
RESET	0.2365	0.2310	**8.1210	**5.2759	F(3,1000)
RESET2	**6.2349	**10.076	**15.940	**7.3196	F(3,1000)
Tsay	1.5008	1.7439	*2.1521	0.8031	F(10,1000)
McLeod-Li	**106.59	**347.76	**332.45	13.792	$\chi^2(50)$
BDS(6)	**187.54	**1081.5	**9891.0	**404.08	N(0,1)
ARCH-LM	**891.10	**895.15	**56.170	0.8562	$\chi^2(4)$
Neural Net	*4.7495	2.3152	0.2514	0.6815	$\chi^2(2)$

Table 5 reports the same tests for the series at a weekly frequency. While the Keenan test again finds no evidence of nonlinearity, the RESET test is now significant (at the 1% level) for the Random Walk and GARCH residuals. The RESET2 test is significant at the 1% level for all four series (at monthly frequency it was significant only for the logs and log-differences). The Tsay test now indicates nonlinearity in the Random Walk residuals at the 5% level (one asterisk). The McLeod-Li test has become significant at the 1% level for the first three series but still reports no nonlinearity for the GARCH residuals. At this frequency, the BDS test also rejects the *iid* hypothesis for the GARCH residuals as well as for the other three series. The ARCH-LM test gives the same results as for the monthly series by rejecting homoskedasticity for all the series except the GARCH residuals. The Neural Network test is now significant at the 5% level but only for the Raw Data series. Overall, the number of significant rejections of linearity has more than doubled compared with the monthly data.

This full-sample analysis may lead us to conclude that a GARCH(1,1) model effectively eliminates any nonlinearities at the monthly level. At the weekly frequency there is much wider evidence of nonlinearity in both the first and second moments and the GARCH(1,1) standardised residuals register significant nonlinearity (at the 1% level) for three different tests, RESET, RESET2 and BDS. Clearly there is more structure here that needs to be explained and for this we turn to the recursive analysis.

5 Recursive Analysis

The tests are now applied recursively from the beginning of the sample, adding one observation at a time and re-estimating the AR(4) linear filter and the test statistics over the observations up to that point. To test the parameter stability of the linear filter, which itself provides a crude check of the adequacy of the linear approximation, the Hansen (1992) test is also run recursively. Figures 4-7 present the results graphically for the series taken at the weekly frequency although the same general pattern of periodic nonlinearity can be seen in the monthly data. In each of the figures, the statistics are divided by their critical value (which is generated separately for each observation as the sample size grows); this means that when the graphs reach the level of 1.0 the statistic has reached the indicated critical level.

For the weekly Raw Data, Figure 4 shows that the recursive Hansen test detects parameter instability in the linear AR(4) in 1982. The Tsay test indicates an isolated period around 8 Nov 1976 which just hovers above the 5% critical level before dropping off and then becoming significant again between 1987 and 1988 although it is not significant for the sample taken as a whole. The McLeod-Li test reveals a sudden rise to the 1% critical level about 19 Oct 1981 and continues to be significant for the rest of the sample. The Neural Network test is significant at two points within the sample around 8 Mar 1976 and again from 19 Nov 1984 to 4 Feb 1985 although it is not significant over the sample as a whole. For the weekly logs; in Figure 5 the Hansen test detects parameter instability in the AR(4) starting around 17 Aug 1981. Recursive analysis reveals a period of significance in the RESET test from about 30 May to 26 Sep 1977 at the 5% level which is again hidden in the full-sample analysis. The Tsay test is significant at the 5% level in full-sample analysis but the recursive analysis reveals several different periods when it is significant at the 1% level: (22 Mar to 26 Jul 1976, 6 Sep 1976 to 10 Oct 1977, 10 Jan to 1 Aug 1977 and 7 Nov 1977 to 1 May 1978). The McLeod-Li test passes the 1% threshold around 5 Oct 1981 and begins a dramatic rise around 18 Mar 1985. For the weekly Log Differences, Figure 6, the Hansen test is increasingly significant starting from mid 1982, indicating parameter instability in the AR(4) filter. The Keenan test is significant at the 5% level at just one isolated point in the sample at 18 Mar 1985. The RESET Test identifies a sustained period of nonlinearity early in the sample from 17 Jul 1978 to 22 Nov 1982 and then a period of effective linearity before a second rise breaking the 1% level again at 15 Apr 1985. The Tsay test identifies two similar periods at the 1% level but the first begins earlier towards 16 Aug 1976 and ends at 20 Nov 1978. The McLeod-Li test becomes significant at the 1% level at 12 Nov 1984 but again takes a sudden leap at 22 Jul 1985. For the weekly GARCH standardised residuals, Figure 7, the Hansen test detects no parameter instability in the AR(4) filter. The Keenan test has an isolated period from 9 Feb 1976 to 3 Oct 1977 during which it is significant at the 1% level. The start and end of this "bubble" correspond with the two negative and then positive outliers in the GARCH residual series. Their effect is also visible in several of the other tests. Intriguingly, the McLeod-Li test moves down on 2 Feb 1976 where the other tests increase. Both RESET tests indicate first moment nonlinearity for much of the sample starting in 1976 and 1977.

We present one final set of results in figure 8 which attempts to correlate the observed evidence for first moment nonlinearity detected in the recursive tests presented above with evidence of volatility in the conditional variances from the estimated GARCH(1,1) process. In the individual graphs shown in figure 8 the horizontal axis shows

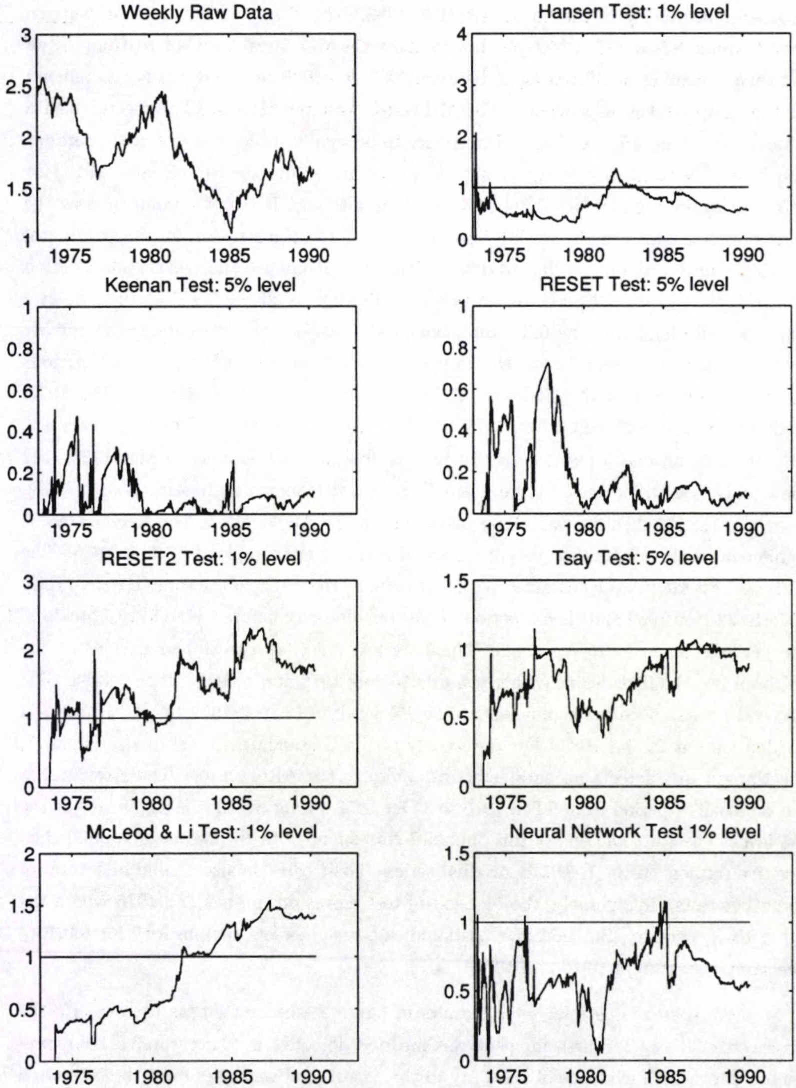


Figure 4: Weekly Raw Data

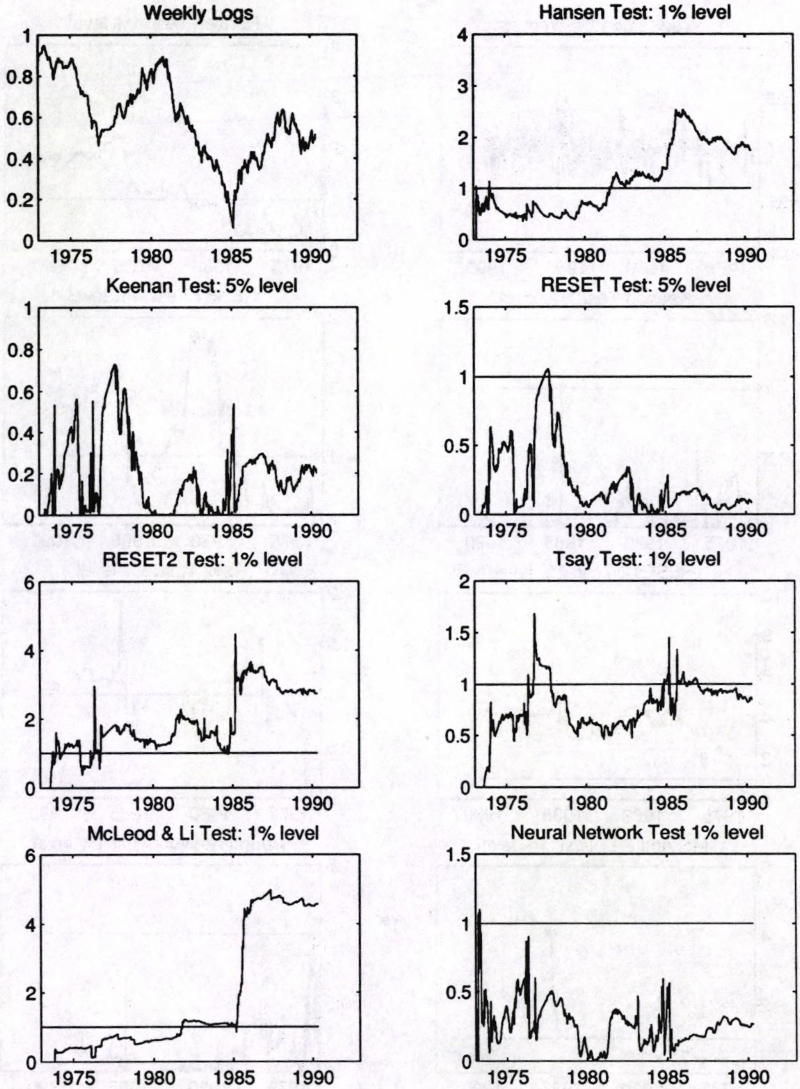


Figure 5: Weekly Logs

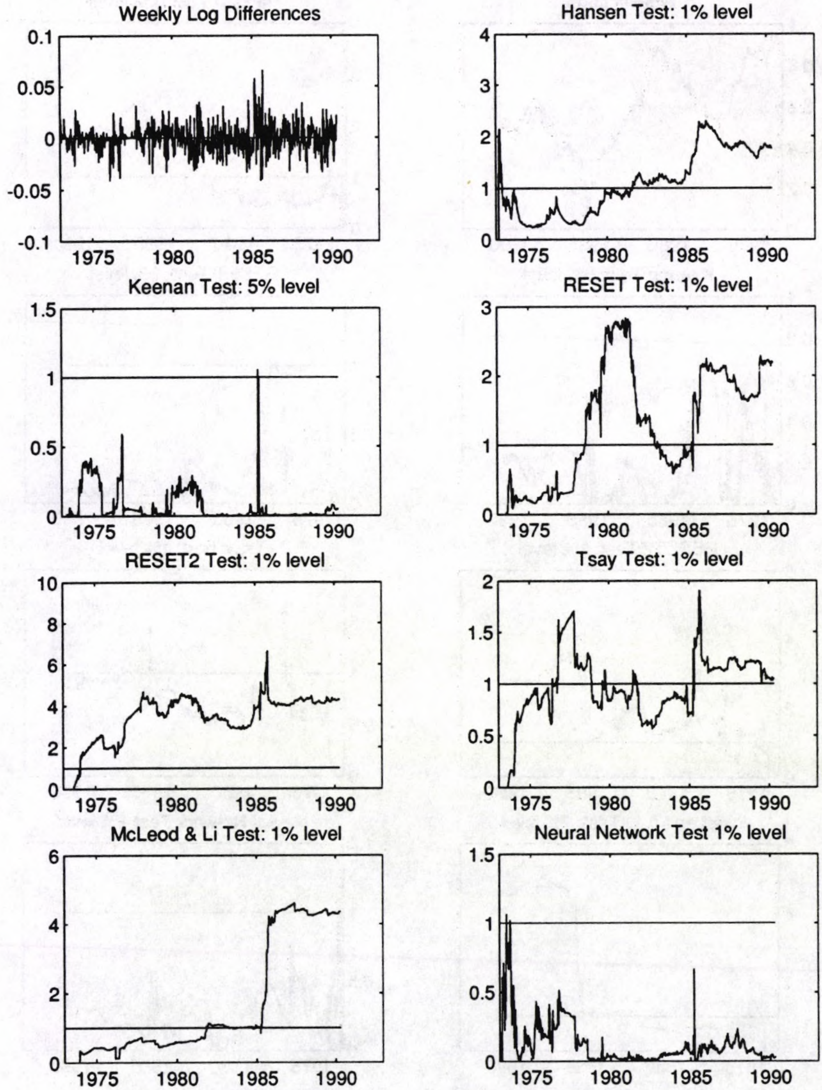


Figure 6: Weekly Log Differences

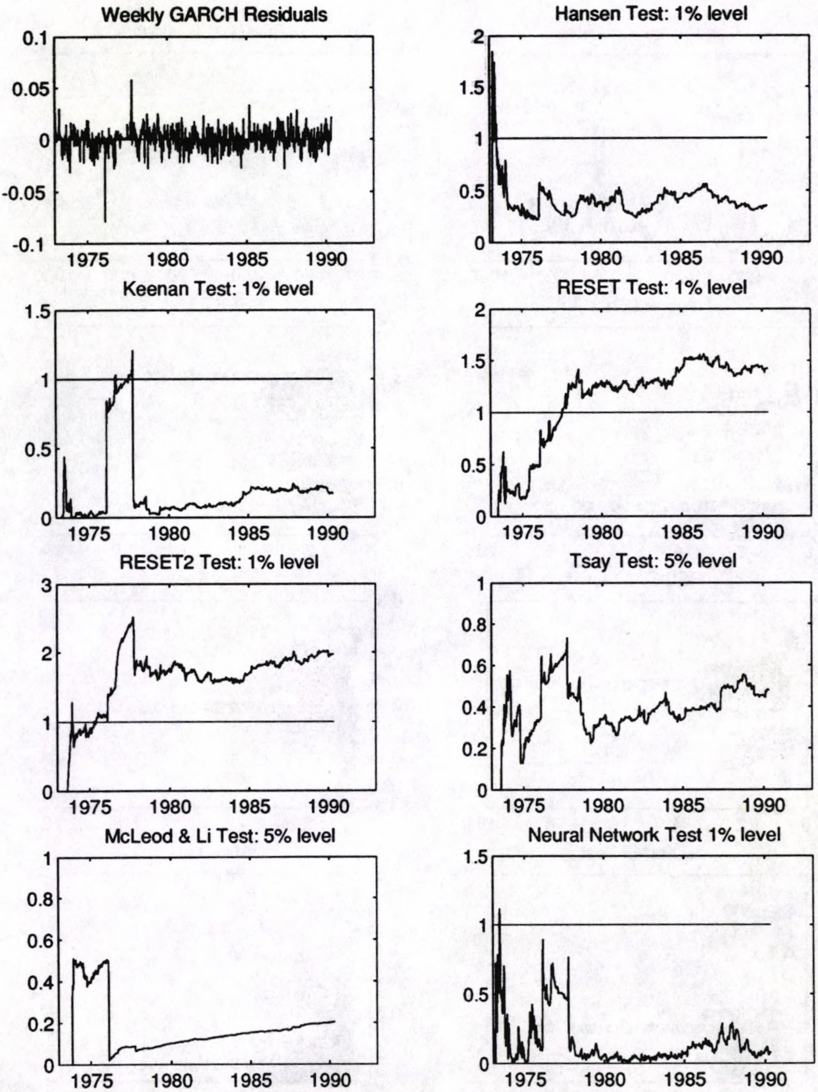


Figure 7: Weekly GARCH Residuals

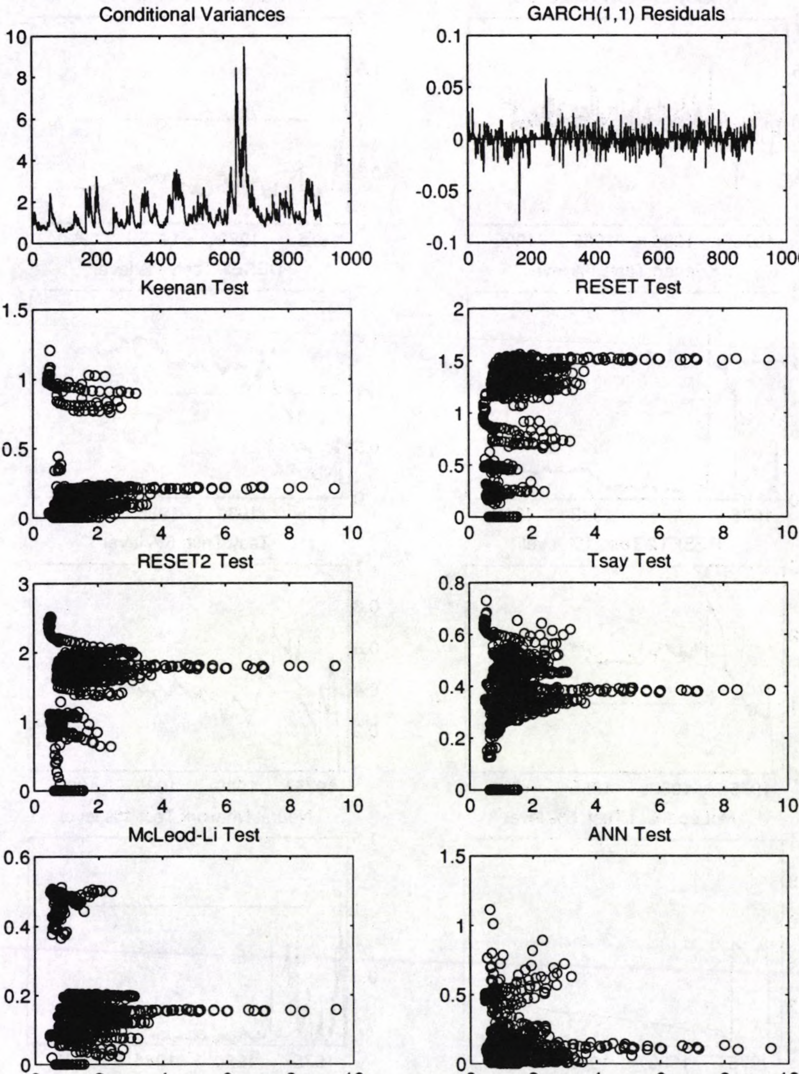


Figure 8: Conditional Variances and First Moment Recursive Tests

the estimated conditional variances and the vertical axes show the indicated recursive first moment test statistics for the log differences series. The lack of association is clearly apparent with significant first moment nonlinearity being associated with periods of both high and *low* volatility. While it may be easy to rationalise high volatility with evidence for first moment nonlinearity what was unexpected was the concentration, particularly with the RESET tests, of significant first moment nonlinearity with low volatility. Clearly our initial presumption that both first and second moment nonlinearity would occur at the same periods is not correct.

Given the different implicit alternative hypotheses that these tests have power against it is difficult to draw precise conclusions from this recursive analysis. However it does seem clear that there are indeed periods within the sample at which first moment linearity breaks down and that these periods do not correspond necessarily with periods of high volatility. The conclusion we then draw is that modelling such data with GARCH-ARCH type models may be masking a more complicated nonlinear structure in the first moment of the data. Hence it may be more appropriate to consider refined nonlinear conditional mean specifications than adopting more complex second moment specifications to model financial data of this sort.

6 Identifying Periods of Nonlinearity

Given the preceding evidence we now attempt to match those points in the sample which proved critical for the recursive nonlinear tests to macroeconomic events affecting the US Dollar/UK Pound Sterling exchange rate. The focus is limited to the log differences of the exchange rate at weekly frequency. This was the horizon at which most nonlinear activity seemed to take place and the log differences transformation is by far the most studied in the empirical literature on exchange rates. First we used the graphics of the recursive nonlinearity tests to pick out the periods when each test moved dramatically, rising through its critical level or dropping back below it. From these observations we traced the corresponding week and checked past issues of *The Economist* newspaper for relevant news at that date. In particular, we looked for news identifying changes in government policy as to intervention in the foreign exchange market, or news that could be interpreted as suggesting a shift in the fundamental determinants of the exchange rate. Of course, if the market is subject to "fads" or speculative bubbles, the appearance or disappearance of nonlinear dynamics may be unrelated to the arrival of any hard "news" but only reflect a spreading misperception in the market or spreading awareness that average market expectations are incorrect.

The results of this exercise are reported in Table 6. Naturally, such an investigation is open to the charge that *ex post* any investigator can always dig up some event to explain why exchange rate behaviour changed in any given week. That is to say that *ex ante*, the "news" cited as an explanation for nonlinear behaviour in a given week may not have been identifiable as such at the time. However, it is striking that just by examining a univariate series in isolation, these statistical tests for nonlinearity were capable of identifying such important events as the end of the fixed exchange rate regime in October 1977, or the 1978 decision by US President Carter to abandon the noninterventionist policy and actively defend the Dollar in October 1978, and the similar U-turn by President Reagan, who switched to a interventionist policy with the decision to stop the dollar's rise at the September 1985 summit of G7 leaders at the Plaza Hotel in New York. These three events suggest that models of discontinuous intervention could successfully exploit nonlinearities identified in the data. On the other hand, there are weeks picked up by the nonlinearity tests for which there is no directly relevant news reported in the press. This suggests that some nonlinearities in exchange rate data are independent of economic fundamentals and might be better explained by market psychology models such as those based on fads, chartists and noise traders, see Banerjee(1992), Scharfstein and Stein (1990), Delong et al.(1991), Frankel and Froot (1986) and Kirman (1993) for instance. In this case, bubbles burst not because of the arrival of some new information but simply because enough agents in the market have learned that average market expectations are mistaken.

7 Conclusions

We may draw four conclusions from the empirical part of this study. First, the comparison of weekly and monthly data even in the full sample analysis indicates that nonlinearity is much more pronounced at the higher frequency. This is to be expected as swings of opinion and speculative forces may dominate the market in the short run while the fundamentals may dominate in the longer run. This is consistent with the contradictory picture of investor behaviour portrayed in MacDonald and Taylor (1992) according to which the market may hold "irrational" short term expectations simultaneously with "rational" long term expectations. The second conclusion is that when sample data is taken as a whole, there is only relatively weak evidence of nonlinearity in the mean. However, the recursive analysis reinforces the picture of separate episodes of significant nonlinear dependence. The third point we can make is that nonlinearity seems to be found even in the residuals of the Random Walk model and the GARCH residuals. Even after filtering these residuals with an AR(4), recursive nonlinearity tests exceed their critical levels at several points within the sample. This implies that the superior out-of-sample forecasting performance of the Random Walk relative to structural exchange rate models is likely to be uninformative as it is based on the comparison of two misspecified models. It also suggests that any well specified model of exchange rate determination must account for this observed periodic nonlinear dependence which the GARCH specification fails to pick up. We have also found, admittedly to our initial surprise, little association between those periods indicating first moment nonlinearity and periods of high volatility suggesting that if GARCH processes are being used to approximate first moment misspecification then perhaps it may be more appropriate to consider respecifying the conditional first moment before formalising our ignorance by fitting a GARCH model to the second moment.

We have also emphasised that if economically meaningful behaviour is restricted to approximately linear parts of the phase space then nonlinearity in the first moment may be difficult to detect in economic data using standard methods although it is important to recognise that questions of scale, both in terms of units of time and in the relative range of the state variables' variation are critical in deciding whether linear approximations are likely to be adequate. Conditional inference may then provide a more relevant statistical framework for the detection of nonlinearity than the unconditional approach currently adopted.

Footnotes:

1. Much of the argument and material in this section has been drawn from Salmon(1993). Reference may also be made to Barndorff-Nielsen and Cox (1994) for a more detailed discussion of conditional inference.

2. Cox (1980), considered how the notion of ancillarity can be extended when exact ancillaries do not exist.

3. Note that the monte carlo evidence of Davidson and MacKinnon (1983) and Bera and McKenzie(1986) regarding the relative slowness by which "observed" forms of the LM statistics approach the asymptotic χ^2 distribution is strictly not relevant to the discussion as to which statistic, conditional or unconditional, should be used for inference. The question of the finite sample distribution of the conditional LM statistic is a separate question.

4. For completeness we repeat the description of these tests which is often taken virtually word for word from Lee, White and Granger(1993).

5. Notice this corresponds to the Thursby and Schmidt (1977) RESET test and not the RESET2 test as specified in Lee, White and Granger (1993).

6. A more complex specification than the GARCH(1,1) was not found to be necessary.

Table 6: Dates of Significant Indication of Nonlinearity

date	Statistics	News
28-Jan-74	Sharp rise in KEENAN (but not significant)	US liberalizes International Capital Flows on 29-Jan-74. The Reserve Requirement on US deposits makes the Dollar cheaper in London than in NY.
16-Jun-75	Sharp drop in KEENAN	Greenspan as chairman of Council of Economic Advisers announces the end of the sharpest US recession since the second world war.
21-Jun-76	Sharp rise in KEENAN (but not significant)	Bank of England fails in attempt to block 4-month dive in Sterling due to liquidity from new Euromarket. US envoys to G5 meeting in Paris criticise the poor state of the UK economy.
11-Oct-76	Sharp drop in KEENAN	Callaghan Government faced with domestic crisis seeks an IMF loan to stem slump in Sterling.
18-Oct-76	TSAY turns significant	UK announcement shows soaring money supply. DM revalued upwards in Snake.
31-Oct-77	TSAY drops sharply but remains significant	Sterling FLOATS as Bank of England abandons intervention. Expected relaxation of exchange controls does not appear in Healy's mini-budget.
30-Oct-78	TSAY drops to below significance level	Following poor reaction to economic programme, Carter takes strong action to defend \$: Fed doubles swaps with other Central Banks. US borrows SDRs from IMF for first time.
10-Sep-79	Sharp rise in KEENAN (but not significant)	Basle meeting of Central Bank governors chooses not to realign EMS. Thatcher U-turn on selling North Sea oil, postponing privatisation of BP.
10-Aug-81	Sharp drop in KEENAN	Reagan signs record tax break and sacks air-controllers. \$ and Yen soar. Bank of England suspends MLR, abolishes Reserve Asset Requirement. and cuts banks' cash ratio.
18-Mar-85	Peak in KEENAN (5%) and in McLEOD-LI (1%)	Ohio Savings and Loan failure prompts bursting of \$ bubble. Volcker announces Fed is ready to lend. Moderate tax cuts in Lawson budget please forex market. Pound soars.
1-Apr-85	Sharp drop in RESET	US announces faster money growth than expected. Volcker indicates \$ may fall.
8-Apr-85	Sharp drop in KEENAN	Revised US GNP figures prove poor. Huge US trade deficit (\$11.4bn) announced.
15-Apr-85	RESET2 & TSAY rise	Dollar recovers despite Securities & Exchange Commission closing of a New Jersey trader.
29-Apr-85	Sharp drop in RESET2	France obstacles GATT at Bonn G7 summit. British Gas privatisation announced.
5-Aug-85	Sharp rise in RESET2	US announces record \$13.4bn trade deficit as Congress passes disappointingly big budget.
16-Sep-85	Sharp drop in RESET2	EMS entry speculated as UK launches \$2.5bn floating rate note issue to rebuild reserves.
23-Sep-85	Peak in TSAY	Plaza meeting. Reagan performs U-turn abandoning non-interventionist stance to organize concerted effort to drive the Dollar down. For the first time, US becomes net debtor.
24-Feb-86	Sharp drop in TSAY	Federal Reserve and US Treasury disagree publicly on Dollar. Oil prices fall.

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